Gravity Compensation of a 6-RSS Parallel Platform

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CERTIFICATE

This is to certify that -

-Aneesh Sinha(18JE0106) ..………………..

- Akash Das(18JE0057) …………………

- Ishan Kumar(18JE0364) …………………

has satisfactorily completed the project entitled "Gravity Compensation of a 6-RSS Parallel Platform" under the guidance of Prof. Arun Dayal Udai. This Project report is submitted to the Department of Mechanical Engineering of Indian Institute of Technology (ISM) Dhanbad in partial fulfillment of the requirement for the award of the degree of Bachelor of Technology in Mechanical Engineering during the academic year 2021-2022

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INTRODUCTION

Due to their advantages over serial manipulators, such as superior rigidity, high load-to-weight ratio, high speed, and precise positioning, parallel manipulators have attracted considerable interest in the previous four decades.

One of the most popular parallel manipulators is the 6DOF Stewart-Gough mechanism proposed by Gough for tire testing and by Stewart to simulate flight conditions by generating general motion in space. This mechanism contains a moving platform and a base, connected to each other by six length-controlled limbs making the platform manipulator a fully parallel-actuated mechanism with 6 DOF. Therefore, many researches have been held on this mechanism in the context of parallel robotics.

The Stewart-Gough mechanism, which was proposed by Gough for tyre testing and by Stewart to replicate flight circumstances by creating general motion in space, is one of the most common parallel manipulators. The platform manipulator is a fully parallel-actuated mechanism with six degrees of freedom, comprising a moving platform and a base connected by six length-controlled limbs. As a result, several parallel robotics studies have been conducted on this technique.

There are two different structures for this mechanism. The 6-SPS (spherical-prismatic-spherical) structure, also known as the prismatic Stewart-Gough mechanism, is the first and most popular. It uses six hydraulic actuators to provide translational motion of the limbs. The revolute Stewart-Gough mechanism, also known as the 6-RSS (revolute-spherical-spherical) or 6-RUS (revolute-universal-spherical) structure, was first described by Hunt as an example of a 6DOF in-parallel manipulator with rotary actuators. Each limb of this structure is made up of a servomotor mounted on the fixed base platform and connected to the moving platform through a crank-rod rotational connection.

Researchers have been interested in the revolute Stewart-Gough mechanism since the 1990s because of its main advantages over the prismatic counterpart, such as better velocity and dynamic characteristics, better load-to-weight ratio of its moving parts because the revolute motors are installed fixed on the base platform, and being cheaper. As a result, this structure was chosen to be studied in this research.

It is critical to have an adequate dynamic model of a manipulator in order to achieve good control performance. The inverse kinematics and dynamics of a 6-URS (universal-revolute-spherical) parallel manipulator have been derived and built using the natural orthogonal complement (NOC) method. The 6-RSS parallel manipulator's inverse kinematics were derived in. In, the inverse kinematics and dynamics of a 6-RSS parallel manipulator were generated using a single link approximation with an even mass distribution on the moving platform and rods on the crank linkages enabling a faster inverse dynamic model derivation. Each of the cranks in the model derived in this reference bears the weight of one rod and one-sixth of the mass of the moving platform. This approximation, however, is only accurate when the moving platform is parallel to the base platform and its origin is in place (xp=yp=0). The similar structure was considered in, where the manipulator's inverse dynamic model was obtained using the decoupled dynamic model approximation.

We use the force distribution algorithm (FDA) to calculate the forces exerted by the upper sections (the moving platform and connecting rods) on the cranks in this study to introduce a novel decoupled inverse dynamic model (IDM) of the 6-RSS Stewart-Gough parallel manipulator. To put it another way, we present a decoupled inverse dynamic model with an unequal mass distribution on the moving platform and rods on the crank linkages. The forces applied by the moving platform and the rods on the cranks in this model change depending on the position of the moving platform and are not assumed to be constant regardless of the position of the moving platform as in.To get the interaction forces between the robot's legs and its environment, the force distribution method has been widely employed in legged moving robots. This approach has also been used in cable-driven parallel robots, but to our knowledge, this method has not been used in parallel manipulators of this type. The static equilibrium equations were derived in our work to acquire the reaction forces, and QR factorization was used to obtain a suboptimal solution for them. The external forces applied to the crank-motor mechanism for each leg of the manipulator were then analyzed.

In this paper, we propose a novel decoupled inverse dynamic model with uneven mass distribution of the moving platform and rods on the cranks employing the Force Distribution algorithm. We call the first model as ‘decoupled inverse dynamic model’ in the sense that each limb can be modeled separately such that six motor-crank models are obtained for the six limbs. It is worth noting that this model is still coupled in fact but it is formally decoupled, since the influence of the motion and the rest of the manipulator is latent in the forces exerted on the cranks by the other moving parts of the manipulator (the moving platform and the corresponding rod) as will be explained in detail later.

LITERATURE REVIEW

Gravity has been taken into consideration in almost every other engineering field from rockets, airplanes, civil engineering to robotics. Existing literature provides us with strong solutions to common problems in compensating gravity in robotics like in the paper “Gravity Compensation in Robotics” by Dr. Vigen Arakelyan Arakelian, typical gravity compensation solutions are systematized and their effectiveness is considered. The above paper encompasses the various techniques applied to compensate gravity but it does not aim to elaborate on the implementation of such techniques on specific robotic systems. Although these discuss gravity compensation they do not dwell deep into its implementation on actual models in robotics.

There are other papers that go into the specific implementations on certain robotic systems like “ AI-Based Gravity Compensation Algorithm and Simulation of Load End of Robotic Arm Wrist Force” by Liang Chen, Hanxu Sun, Wei Zhao, Tao Yu, and “ Algorithm and experiments of six-dimensional force/torque dynamic measurements based on a Stewart platform” by Ke Wen, Fuzhou Du, Xianzhi Zhang. These papers give an insight into the application of specific gravity compensation solutions in different robotic systems.

However, there is still a significant gap in research when it comes to 6-RSS parallel platforms. This paper in effect will fill this gap by analyzing the 3-D model for possible techniques which can be implemented to compensate for gravity thereby increasing payload capacity and optimizing the design.

System description

As shown in Figure 1, this mechanism is made up of two rigid bodies: a moving platform and a base that are connected by six crank-rod limbs. Figure 2 depicts the kinematic chain of one of the six identical limbs of the Stewart-Gough mechanism with revolute actuators investigated in this study, as well as the distribution of the connection points O_{i0} and O_{i6} on the base and moving platforms, respectively.

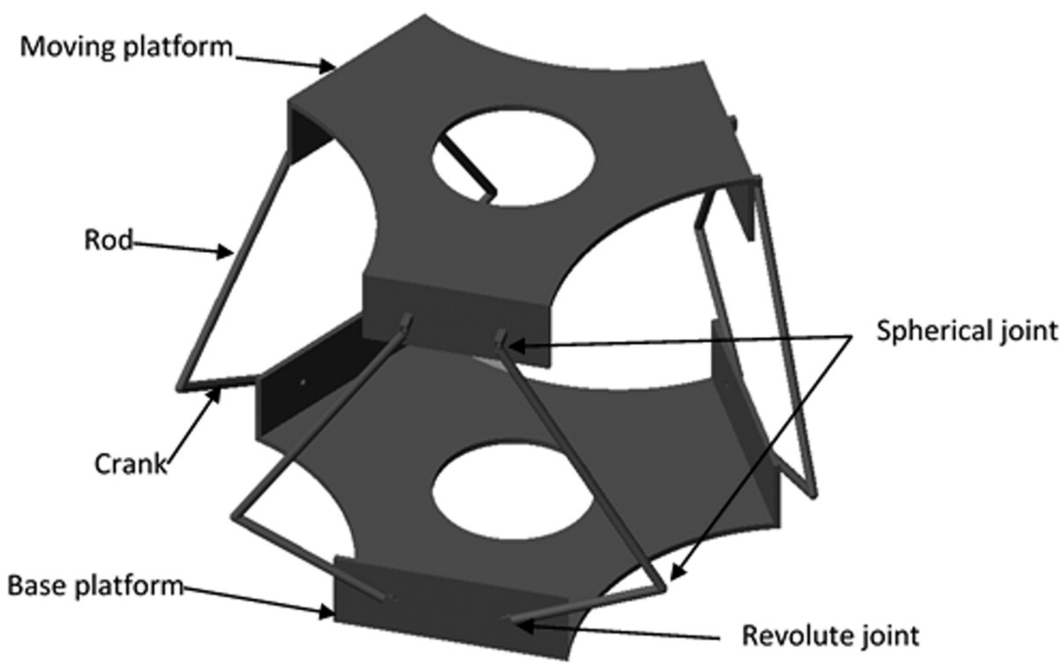


Figure 1. A schematic of Stewart-Gough platform with revolute actuators.

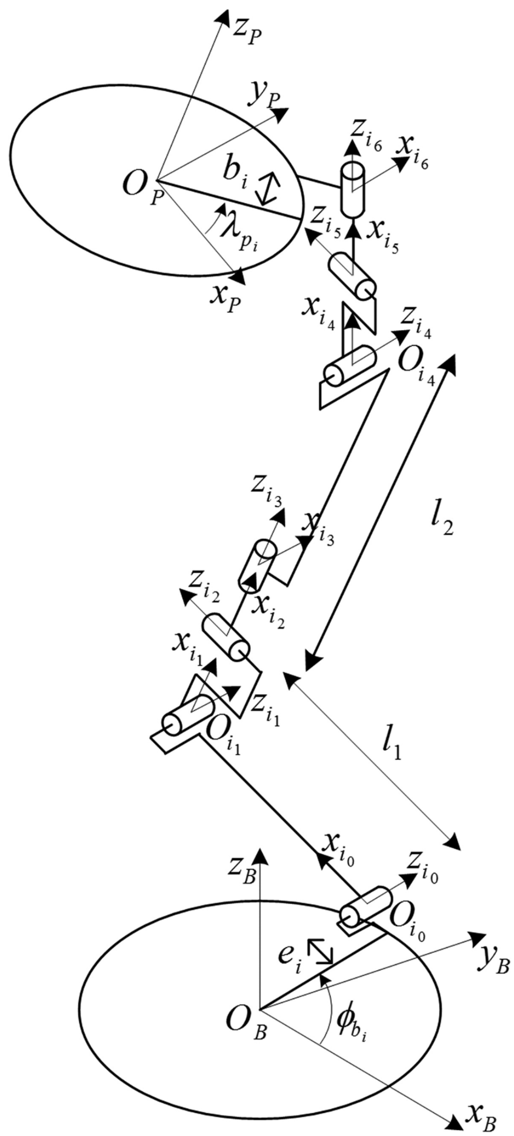


Figure 2. The kinematic chain of the *i*-th limb of Stewart-Gough Mechanism with revolute actuators.

In this figure, O_{i1}=O_{i2}=O_{i3} and O_{i4}=O_{i5}=O_{i6}.

The orientation of $\vec{z}_{io}$ is determined by the angle ϕ_{bi} that it makes with the unit vector $\vec{X}_B$, and the position of the origin O_{io} is determined by the distance ei representing the deviation along the direction of $\vec{y}_{io}$. The same applies to the points O_{i6} on the moving platform through the angle λ_{pi}

and the distance b_i. Each limb is an RSS kinematic chain with the first revolute joint is the active (motorized) one.

To determine the degrees-of-freedom of this mechanism, we apply the Chebyshev–Grübler–Kutzbach criterion:

$$F=\lambda(n-j-1)+\sum_{i=1}^{j} f_i-f_p \quad \textbf{(1)}$$

in which

F= DOF (Degree of freedom)of the mechanism

λ= degrees-of-freedom of the space (λ=6 since our mechanism is a general spatial one)

n= the total number of links in the mechanism, including the base (for this mechanism, n=14).

j= the number of binary joints of the mechanism (j= 12 spherical joints + 6 revolute joints =18)

fi= degrees of relative motion permitted by joint *i*

For spherical joint it is 3 and for revolute joint it is 1

fp= the total number of passive degrees-of-freedom

Substituting in Equation (1), we obtain:

$F=6\times(14-18-1)+(6+18+18)-6=6$

The mechanism explored in this paper has 6 degrees of freedom, according to Equation (1). As a result, it can generate a complete spatial movement with 3 degrees of freedom for position and 3 degrees of freedom for orientation.

Six identical servo motors drive the moving platform's 6-DOF motion. Each servo motor actuates an arm made up of a crank and a rod connected by a passive spherical joint, while the rod is attached to the moving platform by a passive spherical joint. As a result, each limb is an RSS (Revolute-Spherical-Spherical) kinematic chain, with the revolute joint with the base platform serving as the active or servo motor-actuated joint.

Numerical Methodology

The force-balance equations

The weights of the moving platform and the rods, as well as the reaction forces created at joints O_{i1} with the cranks, are taken into account in this study. The mass of the crank is made up of the portion of the rod mass concentrated at its lower end, O_{i1}. The force-balance equations are:

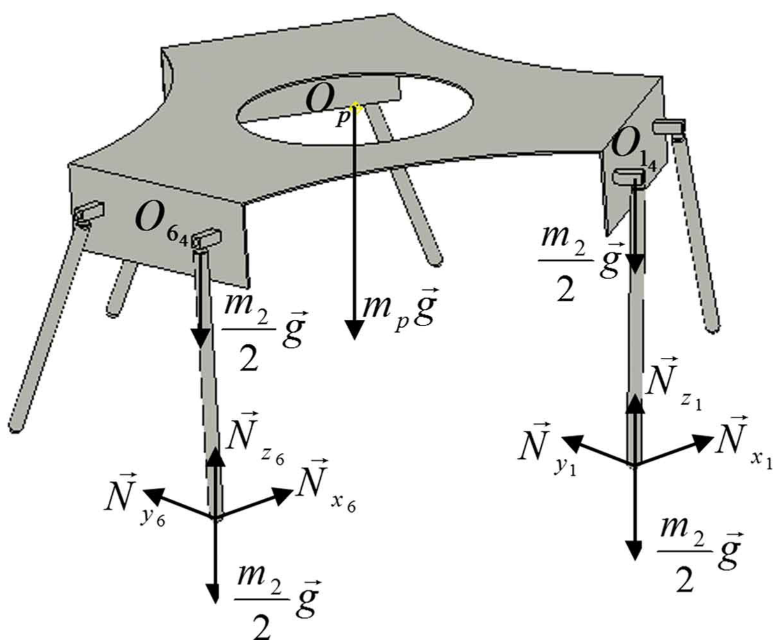


Figure 3. The external forces applied to the moving platform and rods in their interaction with the cranks.

m_p : Mass of the upper platform

m_2 : Mass of the links (all the links are identical so mass will be same)

$\left[N_{xi}\ N_{yi}\ N_{zi}\right]^T$: reaction force vector at the joint O_i, expressed in the stationary *B* frame, whose *x*- and *y*- axes are assumed to be in the horizontal plane and its *z*- axis is vertical pointing upwards.$_i$

m_1 : Mass of the crank.

Force balance equations :-

$\sum F_x = 0$

$N_{x1} + N_{x2} + N_{x3} + N_{x4} + N_{x5} + N_{x6} = 0$

$\sum F_y = 0$

$N_{y1} + N_{y2} + N_{y3} + N_{y4} + N_{y5} + N_{y6} = 0$

$\sum F_z = 0$

$N_{z1} + N_{z2} + N_{z3} + N_{z4} + N_{z5} + N_{z6} - (m_p \times g + 6 \times (m_2 / 2) \times g) = 0$

$N_{z1} + N_{z2} + N_{z3} + N_{z4} + N_{z5} + N_{z6} = (m_p + 3m_2)g$

Writing the equation in matrix format

$$\begin{bmatrix} 0 \\ 0 \\ (m_p + 3m_2)g \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & & 1 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N_{x_1} \\ N_{y_1} \\ N_{z_1} \\ \vdots \\ N_{x_6} \\ N_{y_6} \\ N_{z_6} \end{bmatrix} \tag{2}$$

.

The torque-balance equations about O_p

No torques are transmitted from the rods to the cranks since the crank-rod joints are spherical. As a result, the external torques in the torque-balance equations are equal to zero if these joints are frictionless. In the B frame, the torque-balance equation becomes:

Moment due to point mass m_2 at $O_{i4} = \sum_{i=1}^{6}\left(xO_pO_{i4}\hat{i} + yO_pO_{i4}\hat{j} + zO_pO_{i4}\hat{k}\right) \times \left(-\frac{m_2}{2}g\hat{k}\right)$

Moment due to reaction forces at $O_{i1} = \sum_{i=1}^{6}\left(xO_pO_{i1}\hat{i} + yO_pO_{i1}\hat{j} + zO_pO_{i1}\hat{k}\right) \times \left(N_{xi}\hat{i} + N_{yi}\hat{j} + N_{zi}\hat{k}\right)$

where xO_pO_{ij}, yO_pO_{ij} and zO_pO_{ij} are the projections of the position vector $\overrightarrow{O_pO_{ij}}$ on the *x, y* and *z* axes of the *B* frame, respectively.

The net Moment of the system will be zero. So the equation becomes

$$\sum_{i=1}^{6}\left(xO_pO_{i4}\hat{i} + yO_pO_{i4}\hat{j} + zO_pO_{i4}\hat{k}\right) \times \left(-\frac{m_2}{2}g\hat{k}\right) + \sum_{i=1}^{6}\left(xO_pO_{i1}\hat{i} + yO_pO_{i1}\hat{j} + zO_pO_{i1}\hat{k}\right) \times \left(N_{xi}\hat{i} + N_{yi}\hat{j} + N_{zi}\hat{k}\right) = 0$$

Expanding the equation we get

$$\sum_{i=1}^{6} - yO_pO_{i4}\left(\frac{m_2}{2}g\right)\hat{i} + xO_pO_{i4}\left(\frac{m_2}{2}g\right)\hat{j} +$$

$$\sum_{i=1}^{6}\left(yO_pO_{i1}(N_{zi}) - zO_pO_{i1}(N_{yi})\right)\hat{i} + \left(zO_pO_{i1}(N_{xi}) - xO_pO_{i1}(N_{zi})\right)\hat{j} + \left(xO_pO_{i1}(N_{yi}) - yO_pO_{i1}(N_{xi})\right)\hat{k} = 0$$

This equation can be represented in matrix form as:

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\sum_{i=1}^{6}\begin{bmatrix}0 & -z_{O_pO_{i_4}} & y_{O_pO_{i_4}}\\ z_{O_pO_{i_4}} & 0 & -x_{O_pO_{i_4}}\\ -y_{O_pO_{i_4}} & x_{O_pO_{i_4}} & 0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\-\frac{m_2}{2}g\end{bmatrix}+\sum_{i=1}^{6}\begin{bmatrix}0 & -z_{O_pO_{i_1}} & y_{O_pO_{i_1}}\\ z_{O_pO_{i_1}} & 0 & -x_{O_pO_{i_1}}\\ -y_{O_pO_{i_1}} & x_{O_pO_{i_1}} & 0\end{bmatrix}\cdot\begin{bmatrix}N_{x_i}\\N_{y_i}\\N_{z_i}\end{bmatrix} \quad \textbf{(3)}$$

By performing the matrix multiplication and rearranging the terms, we obtain:

$$\sum_{i=1}^{6}\begin{bmatrix}y_{O_pO_{i_4}}\frac{m_2}{2}g\\ -x_{O_pO_{i_4}}\frac{m_2}{2}g\\ 0\end{bmatrix}=\sum_{i=1}^{6}\begin{bmatrix}0 & -z_{O_pO_{i_1}} & y_{O_pO_{i_1}}\\ z_{O_pO_{i_1}} & 0 & -x_{O_pO_{i_1}}\\ -y_{O_pO_{i_1}} & x_{O_pO_{i_1}} & 0\end{bmatrix}\cdot\begin{bmatrix}N_{x_i}\\N_{y_i}\\N_{z_i}\end{bmatrix}$$

Or,

$$\begin{bmatrix}\sum_{i=1}^{6}y_{O_pO_{i_4}}\\ -\sum_{i=1}^{6}x_{O_pO_{i_4}}\\ 0\end{bmatrix}\frac{m_2}{2}g=\begin{bmatrix}0 & -z_{O_pO_{1_1}} & y_{O_pO_{1_1}} & & 0 & -z_{O_pO_{6_1}} & y_{O_pO_{6_1}}\\ z_{O_pO_{1_1}} & 0 & -x_{O_pO_{1_1}} & \cdots & z_{O_pO_{6_1}} & 0 & -x_{O_pO_{6_1}}\\ -y_{O_pO_{1_1}} & x_{O_pO_{1_1}} & 0 & & -y_{O_pO_{6_1}} & x_{O_pO_{6_1}} & 0\end{bmatrix}\begin{bmatrix}N_{x_1}\\N_{y_1}\\N_{z_1}\\ \vdots\\N_{x_6}\\N_{y_6}\\N_{z_6}\end{bmatrix} \quad \textbf{(4)}$$

By combining Equations (2) and (4), the static balance equation set of the moving platform and rods is expressed as follows:

$$\begin{bmatrix}0\\0\\(m_p+3m_2)g\\ \sum_{i=1}^{6}y_{O_pO_{i_4}}\frac{m_2}{2}g\\ -\sum_{i=1}^{6}x_{O_pO_{i_4}}\frac{m_2}{2}g\\ 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & & 1 & 0 & 0\\ 0 & 1 & 0 & \cdots & 0 & 1 & 0\\ 0 & 0 & 1 & & 0 & 0 & 1\\ 0 & -z_{O_pO_{1_1}} & y_{O_pO_{1_1}} & & 0 & -z_{O_pO_{6_1}} & y_{O_pO_{6_1}}\\ z_{O_pO_{1_1}} & 0 & -x_{O_pO_{1_1}} & \cdots & z_{O_pO_{6_1}} & 0 & -x_{O_pO_{6_1}}\\ -y_{O_pO_{1_1}} & x_{O_pO_{1_1}} & 0 & & -y_{O_pO_{6_1}} & x_{O_pO_{6_1}} & 0\end{bmatrix}\begin{bmatrix}N_{x_1}\\N_{y_1}\\N_{z_1}\\ \vdots\\N_{x_6}\\N_{y_6}\\N_{z_6}\end{bmatrix} \quad \textbf{(5)}$$

or in the form,

B=Ax **(6)**

where the definitions of the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ and the vector $\boldsymbol{x}$ are evident from the above equations.

There are six equations in this set, each with 18 unknowns. As a result, it lacks a singular solution. The QR factorization, as briefly demonstrated below, can be used to solve this underdetermined problem and obtain the minimum-norm solution.

Solving underdetermined equations by using QR factorization

Consider the system $\mathbf{A} \cdot \mathbf{x}=\mathbf{b}$ with $\mathbf{A}(m\times n)$, $\mathbf{x}(n\times 1)$ and $\mathbf{b}(m\times 1)$, where $m<n$. This equation system is underdetermined because the number of equations (m is less than the number of the variables (n). Hence, it has many solutions (if any). However, the minimum-norm solution of this system can be obtained by factoring the Matrix $\mathbf{A}^{\mathbf{T}}$ into the product of an orthogonal matrix $\mathbf{Q}(n\times n)$and an upper triangular matrix $\mathbf{R}(n\times m)$:$\mathbf{A}^{\mathbf{T}}=\mathbf{Q} \cdot \mathbf{R}$ and applying the following algorithm:

1. Solve $\mathbf{R}(1{:}m,1{:}m)^{T}\, \mathbf{z}=\mathbf{b}$

2. Set $\mathbf{x}=\mathbf{Q}(:,1{:}m)\mathbf{z}$

The vector x is the vector of Normal reactions which is used to calculate the troque at revolute joint

The crank-motor dynamic equation

Using the suboptimal weight distribution that resulted from solving Equation (6), the external forces and torques applied to the crank are shown in Figure 4.

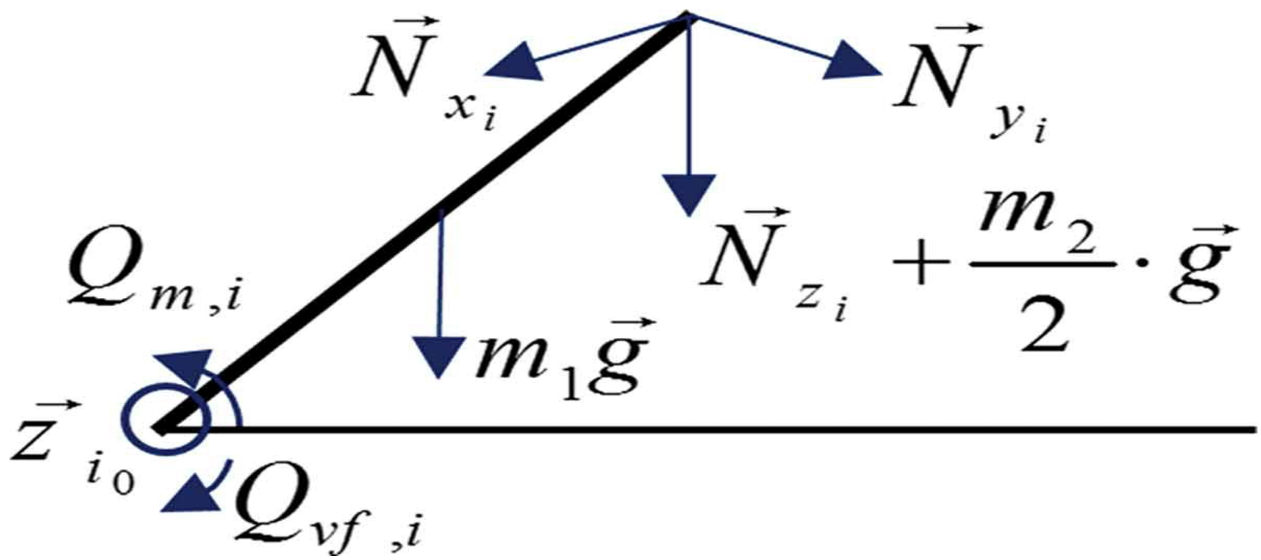


Figure 4. The external forces and torques applied to the crank.

In this model, we consider that the external torques are the torques applied by the motors and the viscous friction torques at the active joints. Hence,

$Q_i = Q_{m,i} + Q_{vf,i}$

The viscous friction is modeled as $Q_{vf,i} = -C_{vf}.\theta_i$, where C_{vf} is the viscous friction coefficient of the active joints, which is considered to be identical for all the active joints. The torque applied by the motors, $Q_{m,i}$ is to be obtained.

Applying Newton's second law for rotation around the axis $\vec{z}_{io}$, which is stationary in the inertial frame *B*, on the assembly of *i*-th crank and the point mass of the rod concentrated at O_{i1} gives:

$$Q_i = \left(I_{1z} + \frac{m_1}{4} l_1^2 + \frac{m_2}{2} l_1^2\right)\ddot{\theta}_i + C_{vf} \cdot \dot{\theta}_i - l_1 \vec{\mathbf{z}}_{\mathbf{i_0}} \cdot \left[\vec{\mathbf{x}}_{\mathbf{i_1}} \times \left(\vec{\mathbf{N}}_{\mathbf{x_i}} + \vec{\mathbf{N}}_{\mathbf{y_i}} + \vec{\mathbf{N}}_{\mathbf{z_i}} + \frac{m_1}{2}\vec{\mathbf{g}} + \frac{m_2}{2}\vec{\mathbf{g}}\right)\right] \quad \textbf{(7)}$$

where I_{1z} is the moment of inertia of the crank around an axis parallel to $\vec{z}_0$ and passing from its center of gravity. If we consider the crank as a cylinder, then $I_{1z} = \left(\frac{1}{12}\right) m_1 l_1^2$.

By combining the equation set (7) into one equation, we can express the decoupled inverse dynamic model in the standard form:

$$\mathbf{Q_{Dec}} = \mathbf{M_{Dec}}\ddot{\theta} + \mathbf{V_{Dec}}\dot{\theta} + \mathbf{g_{Dec}}\,(\theta) \quad \textbf{(8)}$$

in which, $M_{Dec} = \left(I_{1z} + \frac{m_1}{4} l_1^2 + \frac{m_2}{2} l_1^2\right) . I_{3X3}$ is the inertia matrix, $V_{Dec} = C_{vf} . I_{3X3}$ is the Coriolis and centrifugal matrix, and $\mathbf{g_{Dec}}$ is the gravity vector forming the decoupled inverse dynamic model for this manipulator ,where $g_{Deci} = -l_1\left[\left(N_{xi} s\varphi_{bi} - N_{yi} c\varphi_{bi}\right) s\varphi_{bi} + \left(N_{zi} + \frac{m_1 + m_2}{2} g\right) c\theta_i\right]$.

The third term of the right side of Equation (8) accounts for the coupling and nonlinear behavior of the system according to this model, and it can be compensated for by using feedback linearization control.

In our case only static conditions are considered so only gDec is used in the calculation of torque.

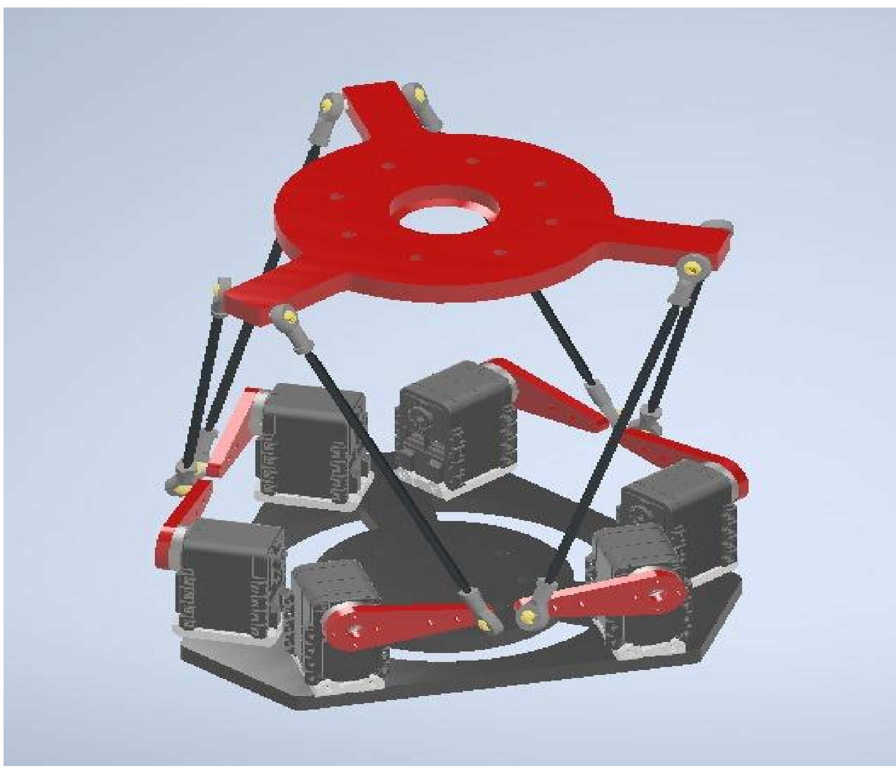
Hence Equation(8) reduces to :

$Q_{Dec} = g_{Dec(\theta)}$

Joint Torque Calculation

Symmetric Position:

In this position, all the joint angles are the same. The result from the calculations also finds the joint torque to be almost the same for all the joints.



Position	Op	O14	O24	O34	O44	O54	O64	O11	O21	O31	O41	O51	O61
X	74.56	60.888	-15.932	91.727	167.361	151.821	-0.632	-2.664	-10.777	152.664	160.777	83.113	66.887
Y	214.275	213.523	214.155	213.451	213.726	213.853	214.21	49.5	49.5	49.5	49.5	49.5	49.5
Z	78.071	174.794	43.109	174.656	42.284	15.646	16.332	132.15	118.097	132.15	118.097	-16.42	-16.42

OpO11	OpO21	OpO31	OpO41	OpO51	OpO61	Sum
-77.224	-85.337	78.104	86.217	8.553	-7.673	2.64
-164.775	-164.775	-164.775	-164.775	-164.775	-164.775	-988.65
54.079	40.026	54.079	40.026	-94.491	-94.491	-0.772

OpO14	OpO24	Op34	Op44	Op54	Op64	Sum
-13.672	-90.492	17.167	92.801	77.261	-75.192	7.873
-0.752	-0.12	-0.824	-0.549	-0.422	-0.065	-2.732
96.723	-34.962	96.585	-35.787	-62.425	-61.739	-1.605

This excel sheet shows the position for the symmetric position. The data from this excel sheet is taken to calculate the **B** matrix .

Matlab Code

```
previoussetting = digits(10);

mp = 0.129; % mass of the upper platform
m2 = 0.013; % mass of the link rods
m3 = 0.004; % mass of the crank
g = 10000;  % acceleration due to gravity (mm/s2)

B = [1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0;
    0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0;
    0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1;
    0 -54.079 -164.775 0 -40.026 -164.775 0 -54.079 -164.775 0 -40.026
-164.775 0 94.991 -164.775 0 94.491 -164.775;
    54.079 0 77.224 40.026 0 85.337 54.079 0 -78.104 40.026 0 -86.217 -94.491 0
-8.553 -94.491 0 7.673;
    164.775 -77.224 0 164.775 -85.337 0 164.775 78.104 0 164.775 86.217 0 164.775
8.553 0 164.775 -7.673 0];

% B matrix formed using current position data (imported from excel)

A = [0; -(mp + 3*m2)g; 0 ; ((-1.605)(m2*g))/2 ; 0 ; ((-7.873)*(m2*g))/2];
```

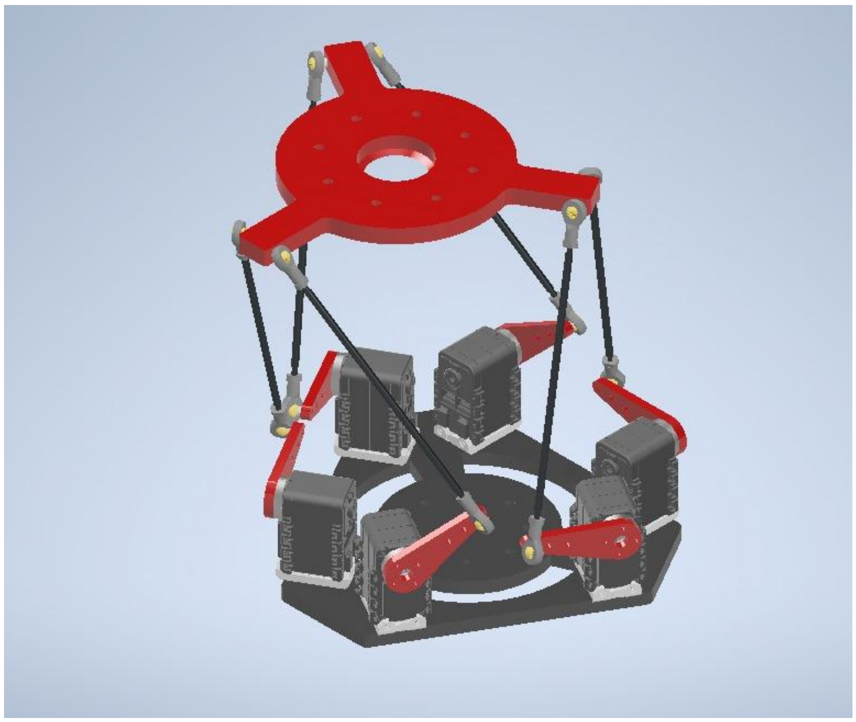
```
[Q,R] = qr(B.');  %QR decomposition

S = inv(R(1:6,1:6).')*A;


x = Q(:,1:6)*S;

theta = [0; 0; 0; 0; 0; 0]; % theta is the angle crank is making with horizontal

j = 2;

for i = 1 : 6

    Fy = -(x(j));

    j = j + 3;

    Fy_net = Fy + (m2 + m3)*g*0.5;  %net force on crank due to platform and links

    torque = Fy_net * 49.40 * cosd(theta(i)); % net torque at each joint

    disp(torque/1000)

end
```

The output of the following code is shown below. The Torque values are applied on the dynamic simulation of Autodesk Inventor of the same model.

```
>> QRDecomposrion_Symmetric_position
   18.0884

   18.0853

   18.0236

   18.0137

   17.9840

   17.9910
```

Unsymmetric Position:-

In this position , the robot's joint angles are different for all the revolute joints. Hence the calculated joint torques are also different for the joints



Position	Op	O14	O24	O34	O44	O54	O64	O11	O21	O31	O41	O51	O61
X	43.74	28.755	-48.065	59.595	135.229	-32.765	119.689	-2.254	-11.107	147.873	160.952	57.26	83.6
Y	225.77	225.487	226.118	225.415	225.689	226.174	225.817	58.327	57.439	78.275	55.287	78.336	56.317
Z	97.972	194.963	63.278	194.825	62.453	36.501	35.815	132.859	117.525	140.447	117.794	-16.42	-16.42

OpO11	OpO21	OpO31	OpO41	OpO51	OpO61	Sum
-45.994	-54.847	104.133	117.212	13.52	39.86	173.884
-167.443	-168.331	-147.495	-170.483	-147.434	-169.453	-970.639
34.887	19.553	42.475	19.822	-114.392	-114.392	-112.047

OpO14	OpO24	Op34	Op44	Op54	Op64	Sum
-14.985	-91.805	15.855	91.489	-76.505	75.949	-0.002
-0.283	0.348	-0.355	-0.081	0.404	0.047	0.08
96.991	-34.694	96.853	-35.519	-61.471	-62.157	0.003

This excel sheet shows the position for the unsymmetric position. The data from this excel sheet is taken to calculate the **B** matrix .

Matlab Code

```
previoussetting = digits(10);

mp = 0.129; % mass of the upper platform
m2 = 0.013; % mass of the link rods
m3 = 0.004; % mass of the crank
g = 10000;  % acceleration due to gravity (mm/s2)

B = [1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0;
    0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0;
    0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1;
    0 -34.887 -167.443 0 -19.553 -168.331 0 -42.475 -147.495 0 -19.822
-170.483 0 114.392 -147.434 0 114.392 -169.453;
    34.887 0 45.994 19.553 0 54.847 42.475 0 -104.133 19.822 0 -117.212
-114.392 0 -13.52 -114.393 0 -39.86;
    167.443 -45.994 0 168.331 -54.847 0 147.495 104.133 0 170.483 117.212 0
147.434 13.52 0 169.453 39.86 0];

% B matrix formed using current position data (imported from excel)

A = [ 0; -(mp + 3*m2)g; 0 ; ((0.003)(m2*g))/2 ; 0 ; ((0.002)*(m2*g))/2];

[Q,R] = qr(B.'); % QR Decomposition of B matrix

S = inv(R(1:6,1:6).')*A;

Q;
R;

x = Q(:,1:6)*S

theta = [10.6; 9.52; 36.83; 6.92; 36.92; 8.16];

% theta is the joint angles each crank is making with the horizontal

j = 2;
```

```
for i = 1 : 6

    Fy = -(x(j));

    j = j + 3;

    Fy_net = Fy + (m2 + m3)*g*0.5; % net force acting at the end of crank

    torque = Fy_net * 49.40 * cosd(theta(i));
    % Net torque acting on the joint

    disp(torque/1000)

end
```

The output of the following code is shown below. The Torque values are applied on the dynamic simulation of Autodesk Inventor of the same model

```
>> QRDecomposition_Unsymetric_position
   27.5344

   27.5442

   11.7807

   12.1125

   11.0868

   11.3416

>>
```

Result

The output from the matlab code was successfully verified in dynamic simulation on the 3-D model of the 6-RSS parallel robot in Inventor.The robot balanced itself keeping a stable configuration under the influence of gravity.

Discussion

There are several approaches for gravity compensation like gravity compensation by counterweights mounted on the links of the initial system, gravity compensation by counterweights mounted on the auxiliary linkage connected with the initial systems, gravity compensation by springs, gravity compensation with auxiliary actuators.

Apart from the decoupled inverse dynamic model approach lagrangian inverse dynamic model method can also be used to find out the joint torques.The difference between the two models is in the gravity vector that expresses the coupling between the limbs and the nonlinearity of the model.

Conclusion

Upon verifying our approach from the Inventor dynamic simulation we conclude that the decoupled inverse dynamic model approach is correct.

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